Sample Assignment #1: Divisibility Problem

All the questions in this assignment will help you answer the following problem:

Problem: Prove $\frac{a(a^2+2)}{3}$ is an integer for all integers a ≥ 1.

Problem #1: Prove that every positive integer can be expressed in the form 3q, 3q + 1 or 3q + 2, for some integer q.

Problem #2: If a is of the form 3q, show that $\frac{a(a^2+2)}{3}$ is an integer.

Problem #3: If a is of the form 3q+1, show that $\frac{a(a^2+2)}{3}$ is an integer.

Problem #4: If a is of the form 3q+2, show that $\frac{a(a^2+2)}{3}$ is an integer.

Problem #5: Using your results from the previous problems, prove the originally stated problem.

Question #6: What type of proof technique is being used here?

Question #7: Under what conditions is this proof technique viable?

Question #8: How would you relate this result to a non-mathematics student? Do not use any mathematical symbols or specific terminology.