Sample Assignment #2: A Harder Divisibility Problem

All the questions in this assignment will help you answer the following problem:

Problem: Determine all integer solutions to the following equation:

$$n_1^4 + n_2^4 + n_3^4 + n_4^4 + ... + n_{14}^4 = 1599$$

Problem #1: If a is even, calculate the result of a^4 mod 16.

Problem #2: Use a proof by cases to show that a(a + 1) is even.

Problem #3: If a is odd, calculate the result of a^4 mod 16.

Problem #4: What conclusion can you draw about the remainder when a^4 is divided by 16?

Problem #5: Looking at the left-hand side of the original equation, determine the possible remainders that can occur if you divide the whole quantity by 16.

Problem #6: What is the result of dividing the right-hand side of the original equation by 16?

Problem #7: Given the answers to problems #5 and #6, solve the original problem, with proof.

Question #8: What was the key creative step in this solution?

Question #9: Should we look at equations like these mod 16? What general principle can we extract from the solution to this problem that might apply to solving other similar equations?

Question #10: Come up with at least 2 analogies that any person (adult) could relate to, to explain the idea of mod. (One analogy would be the number of hours elapse on a clock, after 12 hours, we "wrap around" back to the beginning, just like mod does.)

Question #11: Find at least one practical application of modulo arithmetic and give a summary of it.