Frege's Sharpness Requirement and Natural Language

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On some interpretations of it, Frege's sharpness requirement is problematic. "Sharpness" is a property had by either terms or concepts which the terms stand for. A term or concept is sharp just in case it is objectively truth-evaluable, that is to say, just in case the extension of the term or the associated concept is delimited precisely, so that there is no object in the domain of discourse for which there is no objective fact as to whether it is an element of the extension of that term or concept.[1] In Frege's words only those concepts for which "it shall be determinate, for any object, whether it falls under the concept or not"[2] shall be considered "sharp," and therefore usable within a language that seeks to establish truth. This seems to be a fine requirement within mathematical and logical languages in which the extension of a given concept is determined by the rules of the formal system. In the natural language of our everyday speech, however, there are no fixed rules that determine when a particular object falls under the extension of a given concept. This motivates the plausibility of the view that natural language concepts are rarely, if ever, sharp. Commentators such as Gary Kemp have used this observation to support the argument that "Frege's overall enterprise is not ultimately viable," or alternatively, that "...the sharpness problem (as I shall call it) is not only inevitable but ruinous."[3] The requirement is seen to be ruinous because it cannot seemingly be carried over into natural language contexts without residue.[4] I will argue that Frege's sharpness requirement does have the consequence that our natural language concepts and concept expressions are non-sharp, and therefore not strictly meaningful. But I will not draw from this the conclusion that Frege's enterprise is not ultimately viable. Rather, I will draw attention to the ways the notion of meaningfulness that is entailed by the sharpness requirement is presupposed in that part of our natural language discourse that is truth seeking. In this way I hope to show that the model of truth and meaning that is contained in Frege's sharpness requirement is viable, and accurately describes an important element of our linguistic lives.

Frege's sharpness requirement is in the first instance a requirement on the precision of mathematical and logical concepts. When he wrote the *Grundgesetze*, Frege was polemical about the need for sharp definitions of such mathematical concepts as "greater than," "number," and "equality." He was dissatisfied with the partial definitions these concepts had received in the past, writing that "Piecemeal definition likewise makes the status of theorems uncertain."[5] For Frege this will not do because in his view theorems of the formal sciences must issue in determinate truth or

falsity, and a theorem with an incomplete concept as a constituent cannot do that. He offers an example of this when he writes "Our sentence would be comparable, say, to the sentence 'Scylla had six dragon necks'. This sentence likewise is neither true nor false, but fiction, for the proper name 'Scylla' designates nothing."[6]

Though the focus of Frege's sharpness requirement is on concepts and concept expressions in the formal sciences he is committed to its being the case that all concepts and concept terms, including those of natural language, are unable to factor into a true or false sentence in the strict sense unless they are sharp. This commitment is shown by Frege's disparaging remarks about the natural language proper name "Scylla," in the above referenced passage and others like it such as this one: "Has the question 'Are we still Christians?' really got a sense, if it is indeterminate whom the predicate 'Christian' can truly be asserted of, and who must be refused it?"[7] The implication is that this natural language sentence and others like it do *not* have a determinate sense. This observation offers us a coherent understanding of Frege's desire to replace the incomplete definitions found in natural language treatments of mathematics with his own conceptual notation, and it also squares nicely with Frege's frequent critical comments about the vagueness of natural language in general. In the preface to the *Begriffsschrift*, likening natural language to the human eye, he writes that "as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate."[8]

Now I will articulate for the reader what the sharpness requirement is in greater detail so that we can see where problems associated with it might enter the picture. Frege conceives of certain mathematical and logical expressions as taking a kind of function/argument form.[9] For example, consider the mathematical expression "2 is the positive square root of 4." We can analyze the constituents of this expression in various ways. Frege's analysis is to say that "2" is an argument combined with the function "x is a positive square root of 4." When we examine this expression for its truth-value we see that it refers to the True and that for all other arguments the result is an expression which refers to the False (e.g., "3 is a positive square root of 4 = F").[10] The mathematical concept "positive square root of 4-hood" and similar concepts are paradigmatic cases of strict adherence to the sharpness requirement. This is because for every object in the domain of discourse there is a fact regarding whether or not it falls under such a concept. There is no object for which it is indeterminate whether or not it belongs under the concept. In the expression under review the combination of function and argument yields the truth-value of T only in the case where the argument is the positive integer 2 and F in all other cases. In this sense the function "x is a positive square root of 4" is as sharp as can be.

To reiterate the essentially metaphysical nature of the sharpness problem consider that a sharp concept like "positive square root of 4-hood" will not become non-sharp simply in virtue of

ones inability to identify its true arguments. For example, an innumerate five year old child may not recognize that 2 is the unique argument that yields truth when combined with the function under review, but that doesn't change the fact that the function is sharp.

We can see what sort of disastrous consequences would follow if mathematical concepts were not sharp in this way. The fact that "2" is the only argument that yields a truth value of T when combined with the function "x is a positive square root of 4" is what enables us to assert truthfully that "the positive square root of 4 when added to 5 gives the sum 7." If there were some argument "x" for which it was indeterminate whether "x is the positive square root of 4" were true or false, then the concept "positive square root of 4-hood" would not be a sharp one. This would mean that we could not truthfully assert that "2" is unique in being the argument which when combined with the function under review yields a truth value of T. It would not be true to say that all other possible arguments yield a truth value of F because there would not be a determinate range of true arguments that correspond to the function. So we would have to throw out the function and all of the more complex functions of which it is a part (e.g., "x is a result of adding the positive square root of 4 to 5"). Here we see that all sentences in which a vague concept occurs lack a definite truth-value and cannot therefore be used in any formal deduction or inferential procedure. This shows that sharpness is a requirement on all concepts and functions used in math and logic. If the concept expressions of math and logic were not sharp then those disciplines would fall by the wayside; they would fail to consist entirely of expressions that unambiguously denoted either the True or the False but not both. As Kemp says, "A non-sharp concept sign lacks denotation."[11]

We now must consider whether there are sharp concepts to be found in natural language discourse. Consider the natural language expression "x is a house." For what objects x will this sentence yield a truth-value of T and for which will it yield falsity? It is clearly circular to say simply that all and only the objects which are houses shall make the sentence come out true and all and only the objects which are not houses will make it come out false.[12] Such a claim would be true, but trivially so. To avoid circularity in answering this question we must do something to determine the content of the concept of "house-hood." This, as we shall see, is impossible.

Determining the content of a concept takes the form of discovering the complete and consistent set of necessary and sufficient conditions for inclusion under the concept term. That is to say, a correct specification of the content of a given concept will be a set of necessary and sufficient conditions such that no further conditions need to be listed (completeness), and none of these conditions will lead to a contradiction (consistency) in picking out all and only the objects that fall under the concept in question. In the mathematical case of a few paragraphs back this is possible. We say that an object x belongs under the concept "positive square root of 4-hood" if and only if x is a positive number and x is such that it yields the number 4 when it is multiplied by itself (these are

jointly necessary and sufficient conditions). Here we can see that the concept is sharp, we have the exact specification of all of its conceptual content.

Let me pause for a moment to explain why I say that discovering the complete and consistent set of necessary and sufficient conditions for the inclusion of an object under a given concept is the way in which we determine what the content of that concept is. I do not mean here to say that the failure of any old conceptual analysis of a given concept by insufficiency or unnecessity demonstrates that the concept in question is non-sharp or has indeterminate content. Or what is the same, that the failure of some conceptual analysis or other to close the domain under the concept shows us that the concept is non-sharp. I mean to say that if there is no *possible* set of necessary and sufficient conditions, no *possible* conceptual analysis that could close the domain under the concept or concept expression under review, then that concept and its associated expression are non-sharp. If God himself cannot give the conditions that would close the domain under a given concept, then that concept is non-sharp. This is why I say that we only determine what the content of a given concept is when we "discover" the complete and consistent set of necessary and sufficient conditions for the inclusion of an object in its extension, and not that we determine that content when we stipulate or create such conditions.

We can run a sorites argument in these terms: For "heap of sand-hood" to be a sharp concept there will have to exist a number "x" such that "x" is the minimum number of grains of sand metaphysically required to have a heap. This number of sand grains will be a necessary condition for the existence of a heap of sand. But if there just is no such number "x," then there is no possible conceptual analysis that could specify for us what objects in the universe are heaps of sand. Determining all of the conceptual content of "house-hood" is impossible. Is an object a house just in case it has four walls and a roof? This is sufficient but not necessary, consider an igloo. If this were the specification of the conceptual content of "house-hood" then the concept would fail to denote many of the objects that are houses. Is an object a house just in case it is a thing in which people live and sleep? This specification appropriately denotes igloos and such like, but it also denotes many other objects besides houses (or at least it's a stretch to say that they are houses) like prisons and hotels. The first attempt gave us sufficient but non-necessary content and the second gave us necessary but non-sufficient content. As we seek to discover the correct specification of the content of "house-hood" we will try out ever more subtle and sophisticated conceptual analyses, but we will never perfectly circumscribe all and only the houses with any of them, because a fuzzier object with respect to that definition will always exist.

In this way we see that many (perhaps all) natural language concepts are irremediably non-sharp.[13] It is impossible to learn the definitive set of necessary and sufficient conditions for the inclusion of an object under the concept term such that it would be true to say "yes this is it, this is

what it is for an object to be a house, only these conditions, none less and none more." It is impossible to learn these conditions because no such conditions exist.

There are two related problems for natural language concepts in connection with Frege's sharpness requirement: they lack definite content (vagueness) and they lack a definite extension (equivocality). We can imagine a concept having one but not the other. Consider an island in which all persons are either completely bald or totally covered with hair. Here "baldness" has an exact denotation but it is not because the content of the concept-baldness is non-vague. If a gray area bald person is flown in from the mainland, the islanders will be utterly confused as to whether he is a baldy or a hairy. Even though baldness has a clear denotation in the restricted domain of the island, it is vague when considered in itself. This serves to show that we can isolate the issue of the exactness of a concept's extension from the issue of the exactness of its content. Gareth Evans writes, "…a concept expression may fail to introduce a function which yields a truth value for each object of the domain [b/c some functions] are not everywhere defined."[14] Yes, but it's possible that there be an exact extension of a concept without there being an exact content to that concept.

For Frege it is only required that admissible concepts are univocal with respect to every object, that bivalence of truth-value is preserved for every possible combination of
object and concept for a given concept. But where natural language concepts are concerned we need ideal intensional criteria in the form of necessary and sufficient conditions which must be satisfied by an object for it to count as a member of the set of things that fall under the concept in question. We immediately recognize that it would be false to say of a bicycle "this object is a ham sandwich," and why is this? It is because it is a fact that none (or very few) of the conditions that define "ham-sandwich-hood" are present in a bicycle and vice versa.[15] So even if we try to frame the controversy surrounding the sharpness requirement in terms of a problem specifying for which objects it is true to say that they belong under a given concept term (if we seek to ignore the intensional criteria that group objects under the concept), the crucial issue of conceptual content creeps in as it is that in virtue of which we correctly identify an object as an element of the extension of some concept or other. As we noted, however this content is impossible to determine.

Gary Kemp agrees that these observations show that Frege has a difficulty at the very least in assimilating natural language discourse to the requirements of his semantic theory.[16] He writes that the attempt to close off this difficulty by constructing a metalanguage "...from which to assign objects to singular terms and extensions to predicates" would be to forsake Frege's view of logic "as the universal 'laws of the laws of nature'," a view which commits Frege to the belief that logical laws such as "the law of the excluded middle acquire thoroughgoing factual portentousness; they assert that reality is really like that."[17] We cannot stipulate within a metalanguage what objects shall belong to the extension of a particular concept because we might get it wrong.[18] We might get it wrong

because there are facts of the matter at work; namely the facts concerning whether some object x belongs under the concept F or not (understood to be natural language x's and F's). If it is not possible to determine for some x whether it is an F this has to be because F has vague conceptual content; it fails to make explicit what it is to be a member of the class of things that it denotes. For Frege a deficient concept of this sort would be considered "an inadmissible sham concept."[19]

The continuity of the application of Frege's semantic theory from formal to natural languages is highlighted in a passage taken from *The Foundations of Arithmetic*. In the passage Frege says "Thought is in essentials the same everywhere: it is not true that there are different kinds of laws of thought to suit the different kinds of objects thought about."[20] The laws of thought specify exactly what it takes for a thought to be a true thought. And whether this thought takes as its objects numbers, or tables and chairs, the rules remain invariant. So it is not an available strategy to accept the pernicious consequences of Frege's semantic theory for natural language concepts and at the same time to say "but after all his logical laws only apply to formal languages." No, we must say that the laws apply everywhere, and in virtue of this we must say that our natural language discourse, consisting as it does of expressions containing non-sharp concepts, is predominantly or wholly non-truth functional.[21]

So almost if not all natural language concepts are non-sharp which means all sentences in which they appear cannot have a definite truth value. The puzzle consists in the fact that we seem perfectly well able to deploy these concepts in our use of language and even to speak truthfully and falsely when we use them. Let's return to the example of baldness from a few paragraphs back. In the world in which all persons are either completely hairless or totally saturated with hair follicles it seems uncontroversial to say of hairless Joe that it is true that he is bald. On the other hand it seems uncontroversially false to say of Joe "it is not the case that Joe is bald." But this contradicts the spirit of the present analysis of Frege's semantic theory as well as his explicit assertion that "Bald people for example cannot be enumerated as long as the concept of baldness is not defined so precisely that for any individual there can be no doubt whether he falls under it."[22]

It seems that we are now in a position to say that Frege's semantic theory and in particular his sharpness requirement for concepts render the majority if not the sum of our natural language discourse meaningless. To be clear this is because "a concept expression is meaningful only if it has a sharp boundary…[and]…the failure to display sharp boundaries is almost universal among the concept-expressions of natural language."[23] But if this is entailed by Frege's semantic theory and if it appears clearer to us than the truth of his theory that we do say things truthfully and falsely even in the presence of non-sharp concepts, then a reductio argument can yield the conclusion that Frege's semantic theory and the sharpness requirement in particular is false.

Let me expand the structure of this reductio to make it explicit:

Premise 1: The sharpness requirement is an essential part of Frege's semantics.

Premise 2: The sharpness requirement says that non-sharp terms can't be used in true sentences. (Sentences containing non-sharp terms are neither true nor false but meaningless.)

Premise 3: "Joe is bald" is a true sentence.

Premise 4: "Bald" is a non-sharp term, and is used in "Joe is bald."

Premise 5: By 2 and 4, "Joe is bald" is neither true nor false but meaningless.

Premise 6: But "'Joe is bald" is true (Prmise 3).

Conclusion: Since the truth of Premise 3 is more well-motivated than the truth of Premise 2, Premise 2 is false. And since the truth of Premise 2 is essential to Frege's semantic theory (Premise 1), Frege's semantic theory is also false.

This is the argument that I take to be the philosophical justification for the above referenced claim that Frege's sharpness requirement is "ruinous."

There appear to be two options available to one who interprets the consequences of Frege's semantic theory in the way that I have. The first is to accept his semantic theory and consequently hold that our natural language discourse is mostly or totally meaningless (reject the literal truth of sentences like that found in Premise 3). The second would be to reject Frege's semantic theory and to suppose that in natural language contexts we do speak meaningfully in a wide class of cases (affirm the literal truth of sentences like those found in Premise 3). I propose a version of the first option with the caveat that expressions in natural language can be regarded as true in a context insofar as the individuals using the concept expression assign to it the same content. Further, I believe that a linguistic community is guided in its assignment of conceptual content by the presupposition that the concept under review has an ideally sharp characterization, which is used as the standard of approximation.

Consider the case of an ordinary conversation between two people who disagree over the truth-value of the proposition "Travis is bald." Mark thinks that this proposition is true and Tom thinks that it is false. Let's say that Travis has a growing bald spot at the back of his head but that he still has a decent amount of hair extending all the way to his hairline. Why do Mark and Tom disagree? They disagree because they have different ideas about the content of the concept "baldness." If they had the same notion of what it means for an object to be a bald object and they were presented with the same data (i.e., it is not the case that only one of them sees the bald spot) then they could not fail to make the same judgment regarding the truth value of the proposition "Travis is bald."

Provided that Mark and Tom have a sufficiently robust notion of what is at stake in their disagreement (i.e., it is not a conflict of baseless opinion but a matter of fact whether Joe is bald), they will work to convince each other that theirs is the notion of baldness that more closely conforms to the objective conditions contained in an ideal specification of baldness. No doubt Tom will say something like "Seriously Mark, a bald spot on the back of the head does not make a person bald. Now if his hairline were to recede an extra inch or two then we could make a case for his being bald but not now." Upon hearing this Mark might either say, "Well, if that's what you mean by baldness then I suppose that he isn't bald," or "No, no…" and then proceed to give his own characterization of what it means for an object to be bald.

Importantly, Mark and Tom do not quit the conversation when they reach a disagreement. They do not throw their hands up in the air and say "Well after all there's no fact of the matter about whether Travis is bald," or "Well I've got my definition of baldness and you've got yours and that's the end of it."[24] Rather they each believe that there is a fact regarding whether Travis is bald and they each believe that theirs is the better analysis of the concept of baldness. The deficiencies or surplus criteria that they ascribe to their counterpart are noted in virtue of the belief that they are in factual error about their judgment and hence in factual error about their analysis of baldness. The belief that there is such a thing as factual error with respect to a judgment of an object's inclusion or exclusion under a concept can only come off if one believes that there is an ideal content to the concept under review. This presupposition of ideal content can be thought of as the belief that ideally the concept of baldness is sharp, and some people get closer to the ideal than others do.

For instance somebody who asserted that the wolf-boy was bald would be judged by the linguistic community to have an extremely deficient notion of what it means for an object to be a bald object. On the other hand Mark and Tom judge each other to have mutually deficient notions of baldness, but nowhere near as deficient as the one who says that the wolf-boy is bald. This judgment of relative proximity to the correct notion of baldness exposes a shared presupposition that there is an ideal standard against which our particular understandings of concepts are judged. This can be thought of as the presupposition that in fact concepts are sharp but that we may err in our approximations to that sharpness.

To make it clear, in the above example featuring Mark, Tom, and Travis, I am attempting to give an atomic case of a community wide phenomenon. Individual exchanges of this kind on a wider scale are what constitute the linguistic life of a community as it seeks ever greater precision in the use of its terms. Some uses seem deviant, others seem in conformity with accepted usage, and future revisions are always possible, but through it all the assumption of sharp content persists. It is the assumption in virtue of which we take ourselves to be saying true and false things everyday.

Allow me to ward off a potential criticism of this argument. If we have the following structure in place: an ideal conceptual content of baldness, and two particular approximations to it then we will have two uncertainties that may destroy the ability of Mark and Tom to communicate about a common subject matter.

The first uncertainty concerns our lack of decisive knowledge that Mark and Tom in fact hold the same notion of baldness even after they discuss the issue and recite agreement. Perhaps Tom's notion of the crucial term "hairline" does not exactly correspond to Mark's. So they could agree that if baldness means in part "substantial receding hairline," that Travis is not bald. But their agreement here might be accidental if they do not assign the same meaning to "hairline."

Another potential worry is that even if we grant that they have exactly the same notion of baldness after their agreement, they cannot presume to jointly know the ideal conceptual content of baldness. Since they both presume that there is a fact about what baldness means which is precise enough to tell us whether any object is bald, there is a great likelihood that neither of them has this ideal notion even after they have achieved agreement about what baldness means. Even if they succeed in judging the facts concerning Travis's baldness correctly we should expect there to be an even fuzzier case which their current notion could not help them to adjudicate. How do these observations impact my analysis?

As regards the first point it must be noted that it is not a requirement on the picture that I am offering that participants in this sort of linguistic exchange have identical understandings of a given concept term. In a letter to Peano, Frege writes that, "The task of our vernacular languages is essentially fulfilled if people engaged in communication with one another connect the same thought or *approximately* the same thought with the same proposition."[25] If it is possible to have even an approximately similar understanding of the concept term then we can form a basis upon which to say what we mean when we say to others that an x is an F. To refute this one would need to argue the stronger claim that it is not possible to have even an approximately similar understanding of a given concept. This would be a kind of linguistic solipsism. It seems to me that the burden of proof is on the person who is concerned to argue that point.

For example, say we agree that "Elvis is dead" is true, but only because we both take "dead" to mean "without a heartbeat." However, I think of "heart" as "blood pumper" and you think of it as "organ at center of chest." If these two conceptualizations of "heart" are not similar enough to make it true to say that we agree about the meaning of "Elvis is dead," then we might as well say that people hardly ever communicate at all, that they are usually walled off in their own private conceptualizations from the rest of the linguistic community. I think this is false, and its falsity shows that the conditions on communication are weaker than the conditions on truth-functional discourse, as Frege also believes.

The second point does not affect my analysis of the role that a notion of sharpness plays in everyday discourse. As I said earlier I am committed to the view that Frege's sharpness requirement is coherent and useful even if that means that strictly speaking our natural language discourse is not explicitly truth-functional. This means that the observation that our understandings of natural language concepts are never fully precise even after we decide on their interpretation has no impact on my analysis. It could only be problematic for one who holds that we do indeed come to know the essential characterization of a given natural language concept after we dispute about it but I am not concerned to argue in that way. Incidentally it does not appear to me to be a serious flaw in this interpretation that I commit myself to the view that whenever we use natural language concepts we fall short of ideal truth. If this analysis shows that only mathematical and logical concepts have ideal sharpness then so be it. They are our few exemplars of ideally sharp concepts. But for that we do not say that natural language concepts are not sharp, only that we cannot come to know them in their ideal sharpness.

Some have said that Frege's sharpness requirement for concepts is ruinous and is grounds for rejecting his semantic theory. I do not believe that this is the case. I believe that it is only the assumption of the sharpness of our natural language concepts that enables us to judge others to be in error regarding their use of given concepts. While this falls short of giving truth conditions for natural language sentences I believe that it is an important moral to be taken away from Frege's writings and it offers some insight into the way we operate in our linguistic lives. Once again I do not claim that Frege had a semantic theory for natural language but that we can distill important observations about language in general from his narrower semantic theory. I believe that the present discussion is an instance of such an observation.

Notes

[1] The sharpness problem is then a metaphysical problem, the problem that it seems that there are no objective facts about what objects belong in the extensions of natural language concepts and concept expressions. There is, however, an associated epistemological worry: if it is the case that there are no objective facts about the extension of natural language concepts and concept expressions then of course it will be impossible to *know* what those concepts refer to. This is because "know" is a factive verb and it is not possible to know something that is not the case.

[2] Gottlob Frege, *Collected Papers on Mathematics, Logic and Philosophy*, ed. B. McGuinness (Oxford: Basil Blackwell, 1984) 148.

[3] Gary Kemp, "Frege's Sharpness Requirement," in *Gottlob Frege: Critical Assessments of Leading Philosophers*, vol. IV, ed. M. Beaney and E.H. Reck (Routledge: New York, 2005) 160.

[4] Throughout this paper we will refer to natural and formal languages as distinct things. I will take this distinction to be primitive, but if it is in need of some characterization I will offer this one: natural language is the kind of thing that is used in everyday conversation and prose writing, and formal language is what is used to carry out proofs in the exact sciences (math and logic).

[5] Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. P. Geach and M. Black (Oxford: Basil Blackwell, 1952) 164.

[6] Frege, *Translations from the Philosophical Writings of Gottlob Frege*, 167.

[7] Frege, *Translations from the Philosophical Writings of Gottlob Frege*, 159.

[8]M. Beaney and E.H. Reck, eds., *Gottlob Frege: Critical Assessments of Leading Philosophers*, Vol. IV (Routledge: New York, 2005) 49.

[9] I will treat of concepts and functions in this paper because they are both the subject matter of Frege's sharpness requirement. I note here that for each function there is corresponding concept and vice versa. For any concept "F-hood" or "F-ness" there is a function which can be stated "x is an F."

[10] "Every assertoric sentence concerned with what its words mean is therefore to be regarded as a proper name, and its meaning, if it has one, is either the True or the False" (Frege, *Collected Papers on Mathematics, Logic and Philosophy*, 163).

[11] Kemp, "Frege's Sharpness Requirement," 163.

[12] Frege advises against these kinds of moves himself when he writes that the definition: "'Signs are called white when they belong to white objects…presupposes that we know what the word 'white' stands for" when in fact the word may lack denotation and consequently not stand for anything. "A word may not be defined by means of itself" (Frege, *Translations from the Philosophical Writings of Gottlob Frege*, 162-63).

[13] Perhaps first person concepts and expressions are sharp such as "me-hood," but I do not wish to enter an opinion on this matter. In any case there is no controversy about the vast majority of natural language concepts.

[14] Gareth Evans, *The Varieties of Reference* (Oxford UP: New York, 1982) 12.

[15] At least we take there to be such facts, but more on this later.

[16] When I say "Frege's semantic theory" I mean his general theory about the mapping of functions to the True and the False in the formal sciences of math and logic; a theory meant to establish rules for the admissibility of a function or concept in any formal deduction. Though this account was intended by Frege to apply to functions in the formal sciences, he thought it to be a maximally general account of what makes an expression true or false, and the failure of natural language concepts to meet these standards is what motivates Frege's oft repeated distaste for the imprecision of natural language.

[17] Kemp, "Frege's Sharpness Requirement," 164.

[18] Of course, if there are no facts about what a bike is or is not, then there are no facts to be wrong about. But we are committed by our everyday practice, e.g., "Hey look at that bike!" to the existence of such truth-making facts even if in fact there are no such. More on this later.

[19] Frege, *Translations from the Philosophical Writings of Gottlob Frege.*, 145.

[20] Gottlob Frege, *The Foundations of Arithmetic*, trans. J.L. Austin (Evanston: Northwestern UP, 1968) xv.

[21] By "truth-functional" I mean objectively decidable for truth value across all objects for a particular concept. A concept is not objectively decidable in this way if there are no facts about its truth conditions.

[22] Gottlob Frege, *Philosophical and Mathematical Correspondence*, ed. B. McGuinness, trans. H. Kaal (Chicago: U of Chicago P, 1980) 100. We should keep in mind that though the example I use here of "baldness" is a classically vague concept, the same remarks will hold of less intuitively vague concepts like "bicycle-hood." (i.e., we can't say whether it is true for some x whether "x is a bike" is true unless "bike-hood" is sharp).

[23] Joan Weiner, "Has Frege a Philosophy of Language?" in *Gottlob Frege: Critical Assessments of Leading Philosophers*, vol. IV, ed. M. Beaney and E.H. Reck (New York: Routledge, 2005), 110.

[24] Or if they do this does not serve as a counterexample to my picture, it is a failure of communication.

[25] Frege, *Philosophical and Mathematical Correspondence*, 115. Emphasis mine.

Bibliography

Frege, Gottlob. *Translations from the Philosophical Writings of Gottlob Frege*. Ed. P. Geach and M. Black. Oxford: Basil Blackwell, 1952.

Frege, Gottlob. *The Foundations of Arithmetic*. Trans. J. L. Austin. Evanston: Northwestern UP, 1968.

Frege, Gottlob. *Philosophical and Mathematical Correspondence*. Ed. B. McGuinness. Trans. H. Kaal. Chicago: U of Chicago P, 1980.

Frege, Gottlob. *Collected Papers on Mathematics, Logic and Philosophy*. Ed. B. McGuinness. Oxford: Basil Blackwell, 1984.

Evans, Gareth. *The Varieties of Reference*. New York, NY: Oxford UP, 1982.

Kemp, Gary. "Frege's Sharpness Requirement." In *Gottlob Frege: Critical Assessments of Leading Philosophers* Vol. IV. Eds. M. Beaney and E.H. Reck. New York: Routledge, 2005. 160-76.

Weiner, Joan. "Has Frege a Philosophy of Language?" In *Gottlob Frege: Critical Assessments of Leading Philosophers* Vol. IV. Eds. M. Beaney and E.H. Reck. New York: Routledge, 2005. 97-123.