In Defense of Positive Relevance: A Reply to Peter Achinstein

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Introduction

In Bayesian confirmation theory, the standard account under which a statement is evidence for an hypothesis is positive relevance. That is, E is evidence for H if and only if:

$$\Pr(H|E) > \Pr(H)$$

and a statement is evidence against and neutral with respect to an hypothesis if and only if:

$\Pr(H|E) < \Pr(H)$ and $\Pr(H|E) = \Pr(H)$, respectively.

Thus, for example, the statement "one has purchased a ticket in a million ticket lottery" is evidence for the belief that "one will win the lottery," because $\Pr(H|E) = .000001 > \Pr(H) = 0$.

Since a statement need only incrementally increase the probability of an hypothesis in order to count as evidence, positive relevance demands very little of evidence, which has led to the criticism, found prominently in the work of Peter Achinstein, that positive relevance cannot reasonably operate as an account of scientific evidence because "[s]cientists and others want evidence that h because evidence that h gives a good reason to believe h."[1] In short, in order to provide a good reason to believe H, E must make believing H more reasonable than believing not-H. That is, E is a good reason to believe H if and only if $\Pr(H|E) > \Pr(\text{not-}H|E)$, which is to say if $\Pr(H|E) > .50$. The demand for good reason leads Peter Achinstein to present putative counter-examples to positive relevance wherein he attempts to show that evidence can offer good reason to believe an hypothesis even though the evidence does not increase the probability of the hypothesis. Though, upon a detailed examination of one such counter-example, we shall find that the condition of positive relevance remains fulfilled, which is to say that the evidence in question does not provide a reason to believe the hypothesis—let alone a good reason—because it does not increase its probability. In order to reject his counter-example, however, we must show why, in principle, a statement need not provide a good reason to believe an hypothesis in order to count as evidence for it. To accomplish this we must show why evidence possesses a sense in which it necessarily entails incremental support. Moreover, we must also show that, contra Achinstein, accepting positive relevance does not entail accepting that a statement may count as evidence for both an hypothesis and its negation.

In rebutting Achinstein's criticisms, we will find that his account of evidence is problematic. Indeed, in the end we shall find that because Achinstein imposes a threshold of rational acceptability upon his account of evidence, he imposes a discontinuous structure upon a continuous concept resulting in his misapprehension of a coherent concept of evidence.

Positive Relevance Challenged

Peter Achinstein presents the following as a counter-example to positive relevance:

H = Bill Clinton will win the lottery.
E1 = The New York Times (NYT) reports that Clinton owns all but one of the tickets.
E2 = The Washington Post (WP) reports that Clinton owns all but one of the tickets.
B = Background information that the lottery is fair and the total amount of tickets equals exactly 1000. [2]

Achinstein purports E2 is evidence for H given E1 & B, despite:

(1) $Pr(H|E2 \& E1 \& B) = Pr(H| E1 \& B) = .999$.

That is, he contends that though E2 does not increase the probability of H given E1 and B, we are inclined to consider it as evidence for H. Achinstein here quite obviously appeals to an unstated assumption, viz., that our degree of belief in the veracity of E1 and E2 equal 1. As Sherrilyn Roush notes, (1) obtains if and only if E1 and B make our degree of belief in H2, viz., "Bill Clinton in fact owns all but one of the lottery tickets," certain, which we may represent as:

(2) $P(H2|E1 \& B) = 1$.[3]

However, it is not the case that E1 and B increase H2 to 1, thus (2) is false, and if (2) is false, then (1) is false, and if (1) is false, then Achinstein's counter-example fails to challenge positive relevance. Nevertheless, recognizing the falsity of (2), Achinstein reprises his counter-example and provides two amended versions, the ideal case and the real case.

In the ideal case, if we consider the NYT and the WP as infallible sources of evidence, (2) and (1) are true. If (1) and (2) are true, then Achinstein's initial counter-example holds:

(3) Given the NYT report (E1), the WP report (E2) serves as evidence for H, despite the fact that E2 fails to increase the probability of H.

In the real case, let N stand for "NYT report is always accurate" and W stand for "the WP report is always accurate." Now, given N and W, Achinstein claims E2 is evidence for H:

(4) Given W & N & E1 & B, E2 is evidence for H.

In nuce, given N and W, Pr(H|E2 & W & E1 & N & B) = Pr(H|W & E1 & N & B) = .999, which leads Achinstein to conclude that (4) violates positive relevance. Moreover, in the ideal case (3) likewise violates the requirement for positive relevance. Thus, per Achinstein's counter-example, and despite the falsity of (2) above, positive relevance fails as a necessary condition of evidence.

Positive Relevance Defended

Let us first clarify what Achinstein's counter-example purports to show by first stating what is not in dispute: First, in the real world, strictly speaking, (2) above is false because of the nature of inductive inferences. Second, because (2) is false, (1) is also false. Third, if taken separately, we ought to consider either the NYT report or the WP report as evidence for our belief that Bill Clinton will win the lottery. This intuition is of course endorsed by the positive relevance view: the probability of Bill Clinton winning a 1000-ticket lottery is undoubtedly increased by the NYT report that he owns 999 tickets. Consider the following. Suppose we were confident in the fact that Bill Clinton owns one ticket and we were aware of the relevant background information (B), then our degree of belief in H would be .001. Now, given the NYT report, we would appropriately update our belief in H. We will, however, update our belief in H proportional to our confidence in the truth of the NYT report. This is so because the report would improve our epistemic context in relation to H, that is, it would give us more reason to believe H than we had before. All things being equal, we would in all likelihood measurably increase our degree of belief in H. Thus, the Pr(H|E1 & B) would be greater than the Pr(H|B) = .001, and likewise for the WP report.

Now, having stated the above, it seems odd that Achinstein asserts the falsity of (2) above while at the same time asserting the truth of (4). Indeed, both are real world cases involving the evidential merit of E2 given H, B, and E1. Had Achinstein not, in the ideal case, introduced the further supposition that both the NYT and the WP are always correct in their reports, he would have contradicted himself in asserting (4) and not (2). Moreover, without (3), which is derived from the ideal case, then (4) loses its relevance. Thus, it is the ideal case that we must scrutinize if we are to defeat Achinstein's counter-example.

In the ideal case, he has us assume that the NYT and the WP are infallible. Although important to the case at hand, however, Achinstein does not inform us if we *know* that the NYT and the WP are infallible. It appears that we have only two options: either we do (the known case) or we do not (the unknown case).

If in the known case we read, say, the NYT report first, we would have no need to read the WP report because our degree of belief would have been raised to the highest degree permitted under the possible available evidence. In other words, the evidential value of either E1 or E2 in the known case would at the very least equal or exceed the evidential value of the conjunction of E1 and E2 in the unknown case—that is, insofar as the epistemic value is determined by approximating a degree of belief of .999 in H. If we read the NYT report first, the WP report would become superfluous and thus add nothing to the evidential context. Hence, we also find in the known case that even though (2) obtains (Pr(H2| E1 & B) = 1), (1) (Pr(H|E2 & E1 & B) = Pr(H|E1 & B) = .999) does not, since E2 would be epistemically irrelevant to H.

If we do not know that the NYT and the WP are infallible, Achinstein may correctly assert that the objective Pr(H|E1 & B) = .999, but our degree of belief in H given E1 and B would not, since we may have reason to doubt the reports. To be sure, depending upon an agent's epistemic context, the degree of belief in H given E1 and B may range between .001 and .999, in which case E2 would further confirm H and thus the Pr(H|E1 & E2 & B) > Pr(H|E1 & B), resulting in the fulfillment of positive relevance.

However, as noted above, to reject Achinstein's counter-example in full we must show why the view of evidence which it engenders is mistaken. It is to this task, then, we presently turn.

Threshold Account of Evidence

In *The Book of Evidence*, Achinstein claims that under his view "concepts such as acceptability, having some foundation, firmness, and confidence, in so far as these depend on probability, are 'threshold' concepts. A necessary condition that must be satisfied for a hypothesis *h* to have any acceptability, foundation, or firmness, and before I have any confidence in it, is that *h's* probability exceed some threshold."[4] In that evidence imputes acceptability, foundation, firmness, and confidence upon hypotheses, evidence is also a threshold concept, such that if a statement is to count as evidence for an hypothesis, the probability of the hypothesis given the statement must be greater than or equal to .50, which satisfies Achinstein's demand that evidence provide good reason, since if Pr(H|E) > .50, then the evidence makes H more likely than not-H [Pr(H|E) > Pr(not-H|E)] and thus H is more probable than not-H and thus believing H is rationally acceptable. Insofar as a statement does not make an hypothesis rationally acceptable, it cannot count as evidence for an hypothesis. For instance, just as we have no grounds for rationally believing that one will win a million-ticket lottery without buying a ticket, we likewise lack grounds for believing that one will win the lottery if one purchases one ticket since the probability rises only to .000001. It is not that one has some, albeit small, reason to believe that one will the lottery after purchasing a ticket; rather, there is *no* reason to believe that one will. This poses a difficulty for positive relevance, for if E (the purchase of a ticket) is evidence for H (the belief that one will win the lottery), then E must provide a reason to believe H. However, despite increasing the probability of H (from 0 to .000001), E does not provide a reason to believe H; indeed, it seems our degree of belief in H has not changed at all: we simply have no confidence in H. Therefore, Achinstein argues, it follows that E cannot be evidence for H. More generally, it follows that incremental changes in the posterior probabilities do not affect rational degrees of belief and therefore statements that purport to stand as evidence for H that merely incrementally increase the probability of H are not evidence for H.

Threshold Account of Evidence Challenged

In brief, in order to obtain an account of evidence that avoids counterintuitively assigning degrees of belief to hypotheses with absurdly low probabilities, Achinstein imposes upon his concept of evidence a structure of reasonableness of belief.[5] As Steve Gimbel points out, one may represent the structure as a step function wherein at a range of probability values the reasonableness of an hypothesis remains constant.[6] At various probability values, however, the reasonableness of an hypothesis increases to the next level of rational acceptability. Until determined levels of rational acceptability are met, one may not have grounds for assigning any degree of belief whatsoever.

First, Achinstein's claim that E must make the probability of H greater than not-H (or some determined threshold) in order to count as evidence for H does not at all seem right since this entails that E is evidence for H only when E confirms H to a level at which it is rational to believe H (and by hypothesis irrational to believe not-H). For there are occasions where currently accepted hypotheses meet with anomalies that, though in their accumulative strength are unable to overturn one's rational belief in the hypotheses, nevertheless decrease one's confidence in them (e.g., the anomalous phenomenon of Mercury's perihelion with respect to Newtonian mechanics), and it is not at all clear that Achinstein's account could make sense of these. Conversely, there seem to exist clear cases in which evidence for a theory or hypothesis accumulates gradually over time, first providing some reason to believe, and then progressing to providing better, and, eventually, good reason to believe a theory over and against its competing alternatives. Furthermore, Achinstein would similarly be at pains to account for various exploratory research studies within the social sciences that seek to gather evidence which, though it is too insubstantial to confirm or disconfirm hypotheses, recommends new and potentially fruitful areas of investigation. Exploratory research often cannot by itself present compelling evidence for either the acceptance or rejection of an hypothesis, but these studies, from which further, more in-depth studies arise, also help apportion research funds and the allocation of otherwise scarce resources. One could proffer further examples which incorporate the various sciences, the law, forensics, and not to mention more colloquial experiences that seem to indicate an incremental sense of evidence.

Even if the above criticism is valid and there is a sense in which evidence provides incremental support, Achinstein could argue that this merely shows that there exists an ambiguity in the way in which philosophers and scientists use the term "evidence." Furthermore, even if Achinstein grants that one could identify a consistent sense in which "evidence" is used to mean incremental support, he could argue that one ought to reject such a connotation because of two difficulties, the first of which was identified above: (a) positive relevance compels the counterintuitive assignment of belief in hypotheses with very low but non-zero probabilities, and (b) positive relevance leads one to accept that evidence can both confirm an hypothesis and its negation.

Achinstein's assertion that positive relevance compels the assignment of belief in hypotheses with very low probabilities is of course true. However, the problem arises for positive relevance when one claims that such an assignment is counterintuitive and contrary to the demands that many place upon a concept of evidence. One may partially answer Achinstein by differentiating between "good evidence" and "evidence." If a statement provides good evidence for a belief, then, like Achinstein's account, given the statement the probability of the belief is high enough so as to be rationally acceptable. If a statement provides evidence for a belief, then it merely makes the belief more probable than before the receipt of the evidence. Good evidence, then, would be a special case of evidence, such that if E is "good evidence," then E is also "evidence." It follows easily enough that when a statement does not satisfy the conditions of "good evidence," but yet satisfies positive relevance, then we may identify the statement as being "some evidence."[7] Insofar as scientific practice is concerned, then, one may identify "good evidence" as the most desired form of "evidence" without difficulty. Concerning the counterintuitive implications of positive relevance, one may respond that the confusion arises from equating "good evidence" with "some evidence" when one uses "evidence" to connote the latter, not the former, and that when one makes the necessary and proper contextual distinctions, the confusion dissipates. In other words, Achinstein's account may serve to explicate a desired concept of good evidence but not evidence *per se*. This seems more correct when one considers that evidence possesses an underlying logical structure that obviates Achinstein's charge that positive relevance is deeply counterintuitive.

To this point we should emphasize that every statement stands in some evidential relationship with every hypothesis. That is, every statement is either evidence for an hypothesis or it is not—never neither, never both—and if it is not, then it is either disconfirming or neutral with respect to an hypothesis. Now, consider again the lottery example. If we conform to Achinstein's account, the statement "one has purchased a lottery ticket" is not evidence for the hypothesis that "one will win the million-ticket lottery" because it does not provide good reason to believe that one will win the lottery. Therefore, the statement must either disconfirm or be neutral with respect to the hypothesis. If the former, then Achinstein is in the awkward position of claiming that the purchase of a lottery ticket disconfirms the belief that one will win the lottery; that is, increasing the likelihood of the hypothesis's truth serves to decrease one's confidence in winning. Of course, Achinstein rejects this and instead asserts the statement is neutral with respect to the claim. If the statement is neutral with respect to the claim, then it must be the case that knowing one has purchased a ticket is irrelevant to knowing whether or not one will win the lottery. If E is irrelevant to the truth of H, however, then knowing E is as epistemically insignificant to H as knowing the truth of any other seemingly irrelevant fact with respect to H, say, for example, that Chianti originated in Tuscany. However, it seems rather clear that while knowing the origin of Italian red wines is immaterial to whether one will win a lottery, knowing whether one has purchased a ticket in the lottery is not. To be sure, as Achinstein himself admits, the statement increases the probability that the hypothesis is true, and thus the statement *must* be epistemically relevant. Therefore, since the statement is epistemically relevant to the hypothesis, it cannot be neutral with respect to the hypothesis, in which case the only alternative that remains is that the statement is evidence for the hypothesis, despite the fact it does not make believing that one will win the lottery more rationally acceptable. However, as previously noted, Achinstein asserts that the fact one has purchased a lottery ticket is not evidence for the belief that one will win. Therefore, his account seems to face a paradox, or it seems he would have to assert that the statement is neither evidence for, against, nor neutral with respect to the claim, in which case it is something else. If the latter, then Achinstein will have to tell us what fourth evidential state the statement stands in relation to the hypothesis; a difficult task considering it is not at all clear what that evidential relationship could possibly look like.

Having replied to (a), let us now turn to (b), the claim that if positive relevance is correct, one must accept that a statement of evidence could both confirm and disconfirm an hypothesis. To engender the criticism, let us consider a fair coin flip. The statement, E, "John is flipping a coin" is under positive relevance evidence for the hypothesis, H, that John will observe heads since it increases the likelihood of H from 0 to .50. That is, Pr(H|E) = .50 > Pr(H) = 0. However, the hypothesis that we will observe tails, not-H, is the negation of H, and is equally confirmed by E as is H. That is, Pr(not-H|E) = .50 > Pr(not-H) = 0. Achinstein contends that the fact that a statement confirms H and not-H should lead us to conclude that E confirms neither H nor not-H. At first blush, the criticism is valid, in which case, on pain of contradiction, the Bayesian would of course have to reject positive relevance. Indeed, as any standard textbook on probability will show, the probability of obtaining a heads or tails on a flip is 1, and the probability of either occurrence is .50, so it stands to reason that tails is the negation of heads. However, this is not, strictly speaking, the case. It is true that if the outcome is heads, then we cannot observe tails. However, the context in which E confirms the heads hypothesis (or the tails hypothesis) is not the same one in which E discriminates between the heads hypothesis and the tails hypothesis. In standard lessons of elementary probability theory, when one countenances the probability of observing a heads versus a tails, one does so within a subjunctive conditional clause. That is to say, one considers the probability of observing heads *if one were to flip a fair coin*. When one couches probability assignments in subjunctive conditionals, one already assumes what would be the case given the truth of the antecedent ("if one were to flip a fair coin"). As the proponent of positive relevance will admit, in our example of the coin flip the fact that the coin has been flipped does not discriminate between observing heads or tails; further evidence would be required (e.g., the air resistance of the coin, the velocity of the coin, etc.). However, with respect to contexts in which one is previously not aware of the flipping of a fair coin, the fact that one is now flipping a coin discriminates between observing heads versus, say, observing a five on a die. Imagine an instance in which an observer is presented with a blank television monitor. The observer is informed that the screen will come on and he will observe an event, either a coin flip or a roll of a die. When asked for his degree of belief in observing, H, the event "coin flip: heads," because he knows the Pr(H) = .125, he will, *ceteris paribus*, assign H a corresponding degree of belief. In this context, not-H consists not only of the event "coin flip: tails," but rather not-H is the set of all alternative competing events {the event "coin flip: tails," the event "die toss: one," … the event "die toss: six"}. Upon receipt of E (the fact that a fair coin is tossed), the observer will update his degree of belief accordingly, namely, he will increase his degree of belief in H to .50 from .125 (similarly if H were the event "coin flip: tails").

Clearly, the evidence in this context does not confirm both the heads hypothesis and the tails hypothesis. Rather, the evidence discriminates H from the set of competing alternative hypotheses and thus confirms H with respect to the set of competing alternative hypotheses. Of course, however, as noted above, further evidence is necessary in order to discriminate H from not-H in the succeeding context, that is, where the head hypothesis is H and the tails hypothesis is not-H. Nevertheless, the result remains: positive relevance need not entail that evidence must confirm both H and not-H.

Thus, since (a) and (b) fail as legitimate objections to positive relevance, and since Achinstein's threshold account seems to suffer from serious difficulties, we may conclude that Achinstein's criticism of positive relevance does not succeed.

Conclusion

The above analysis highlights an essential problem with Achinstein's account: he imposes a threshold for rational belief upon his concept of evidence when evidence and standards of rational acceptance are, conceptually, quite distinct. Though evidence provides firmness and support for beliefs, and therefore makes beliefs rational or not, evidence possesses a logical structure that divorces it from a threshold of rational belief. For the positive relevance theorist, the difficulty that faces Achinstein is avoided by the distinction between "good evidence" and "some evidence." Indeed, whatever strengths Achinstein's account possesses positive relevance can adequately explain with the distinction. In brief, one is rational to believe an hypothesis if and only if $Pr(H|E) > .50$. With Achinstein, the positive relevance theorist will agree that, given the toss of a fair coin, it is not rational to believe that we will observe heads rather than tails. However, it does not follow that Achinstein can contend that positive relevance must therefore lead one to the conclusion that evidence can both confirm and disconfirm an hypothesis. The issue of confirmation is inextricably contextual and hence one must pay careful attention to the context in which one claims a fact is evidence for a belief.

Finally, Achinstein's counter-example fails to reveal a counterintuitive implication of positive relevance, but to reject the counter-example more fully, we showed Achinstein's conception of evidence was itself problematic. The demand for a threshold of rational belief is of course necessary; however, to impose a threshold of rational belief upon a concept of evidence is misplaced, for it leads to the difficulties previously identified. Achinstein criticized positive relevance for being too weak, for permitting too much as evidence, but it seems that this is an unavoidable feature of the concept.

Notes

[1] Peter Achinstein, "Four Mistaken Theses about Evidence and How to Correct Them," in *Scientific Evidence: Philosophical Theories and Applications*, ed. Peter Achinstein (Baltimore: Johns Hopkins UP, 2005) 44.

[2] Achinstein 70.

[3] Sherrilyn Roush, *Tracking Truth: Knowledge, Evidence, and Science* (Oxford: Oxford UP, 2005) 179.

[4] Peter Achinstein, *The Book of Evidence* (New York: Oxford UP, 2001) 73, 74.

[5] Achinstein, *The Book of Evidence*, 74.

[6] Steve Gimbel, "Restoring Ambiguity to Achinstein's Account of Evidence," in *Scientific Evidence: Philosophical Theories & Applications*, ed. Peter Achinstein (Baltimore: Johns Hopkins UP, 2005) 53.

[7] Roush 154.

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